

EE421/521
Image Processing

Lecture 2
IMAGE ENHANCEMENT

1



Introduction

2

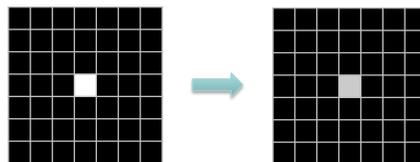
Image Processing Operations

- Spatial domain operations
 - Point operations
 - Neighborhood operations
 - Operations combining multiple images
- Transform domain operations

Point Operations

- Transformation depends on the value of the pixel only, regardless of the values of its neighbors.

$$r(x,y) = T[s(x,y)]$$





Example: Adding A Constant → Brightness Change

- Image has more brightness (larger intensity values) if the additive constant is positive.

Original



+ 50



- 50



By Oge Marques

Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Example: Multiplying with a Constant → Contrast Change

- Image has more contrast (larger range of intensity values) if the multiplicative constant is larger than one.

Original



x 0.7



x 1.4



By Oge Marques

Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Handling Overflow and Underflow

- Handling overflow:
 - Truncate the results to the maximum number allowed
 - Scale(normalize) the results to the maximum range:

$$g = \frac{L_{max}}{f_{max} - f_{min}}(f - f_{min})$$

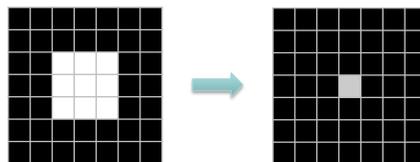
- Handling underflow:
 - Truncate negative values to zero
 - Use the absolute value of the difference
 - Add a positive constant to the difference



Neighborhood Operations

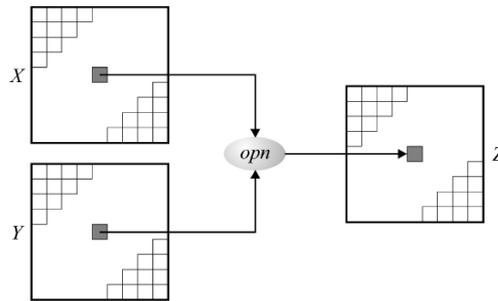
- Transformation depends on the value of the pixel as well as the values of its neighbors.

$$r(x, y) = T[s(x, y) : (x, y) \in W]$$



Operations Combining Multiple Images

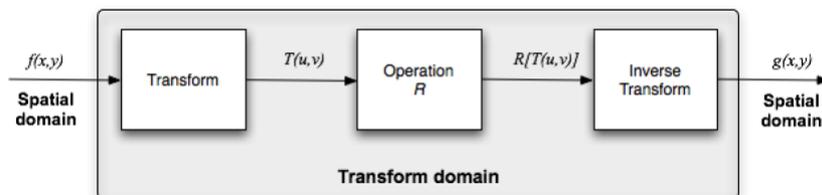
- Two or more images combined pixel-by-pixel, using an arithmetic or logical operator, resulting in a third image



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Transform Domain Operations

- Operations carried out in a transform (e.g., Fourier transform) domain.



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



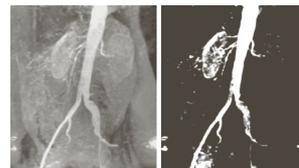
Image Enhancement

11



Image Enhancement Goals

- to improve the subjective quality of an image for human viewing
- to modify the image in such a way as to make it more suitable to further analysis by a human or a computer





Main Techniques

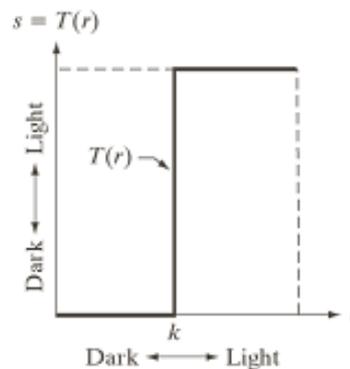
- Intensity transformations
 - Thresholding
 - Intensity-level slicing
 - Contrast stretching
 - Gamma correction
 - Histogram equalization
- Spatial filtering
 - Unsharp masking

13



Thresholding

- r : input intensity value
- $T(r)$: output intensity value
- k : intensity threshold



14

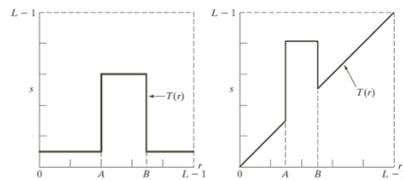
● ● ● | Thresholding



15

● ● ● | Intensity-Level Slicing

A range of intensity levels is highlighted in the output image, while all other values are suppressed or remain untouched.



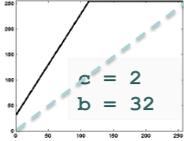
16

Linear Transformations



$s = c \cdot r + b$

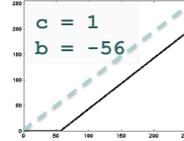
c: controls **contrast**, slope
b: impacts overall **brightness**, y-intercept.



$c = 2$
 $b = 32$



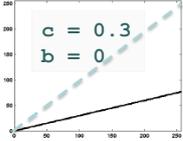
Overall **brightening**,
many saturated pixels



$c = 1$
 $b = -56$



Overall darkening



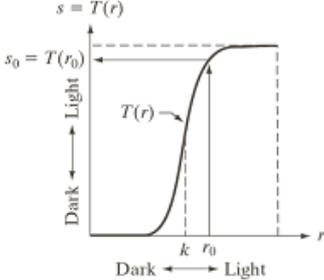
$c = 0.3$
 $b = 0$



Significant **contrast** reduction
Overall darkening

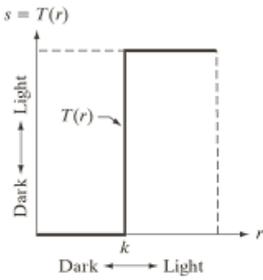
Contrast Stretching

- Small pixel values are compressed toward darker values while large pixel values are pushed toward brighter pixel values.
- Extreme case: binary thresholding



$s = T(r)$
 $s_0 = T(r_0)$

Dark ← Light



$s = T(r)$

Dark ← Light

18

Contrast Stretching using Piecewise-Linear Transformations

Original Image



Adjusted Image



Thresholding vs. Contrast Stretching



Original



Thresholded



Contrast Stretched

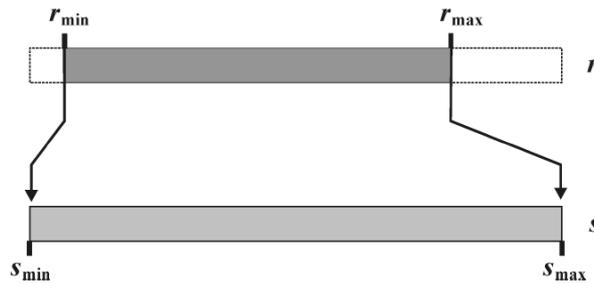
$s = T(r)$

$s = T(r)$



Auto-Contrast Adjustment

$$s = \frac{L - 1}{r_{max} - r_{min}} \cdot (r - r_{min})$$



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Auto-Contrast Example

```
I = imread('salzburg_before.png');
f = double(I);
fmin = min(min(f));
fmax = max(max(f));
g = uint8(255*(f-fmin)/(fmax-fmin));
imshow(g)
```



Original image



Adjusted image

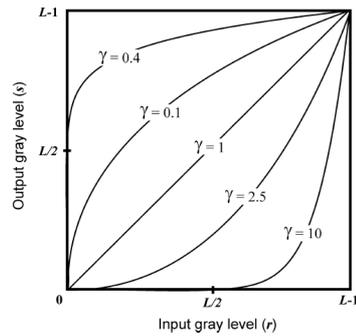
By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Power-Law (Gamma) Transformation

- For gamma < 1, the output image is brighter than the input image
- For gamma > 1, the output image is darker than the input image

$$s = c \cdot r^\gamma$$

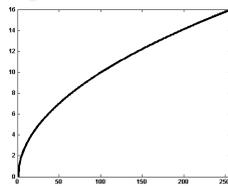


If r is between 0 and 255, choose c such that s is also between 0 and 255. Alternatively, use normalized r and s , i.e., $0 \leq r, c, < 1$, in which case $c=1$.

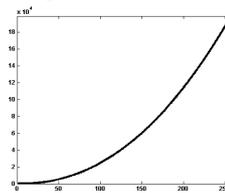


Gamma Transformation

gamma = 0.5



gamma = 2.2



By Oge Marques

Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Gamma Correction

Scanner, printers, and display devices have a power-law response to input intensity:

$$S = r^\gamma$$

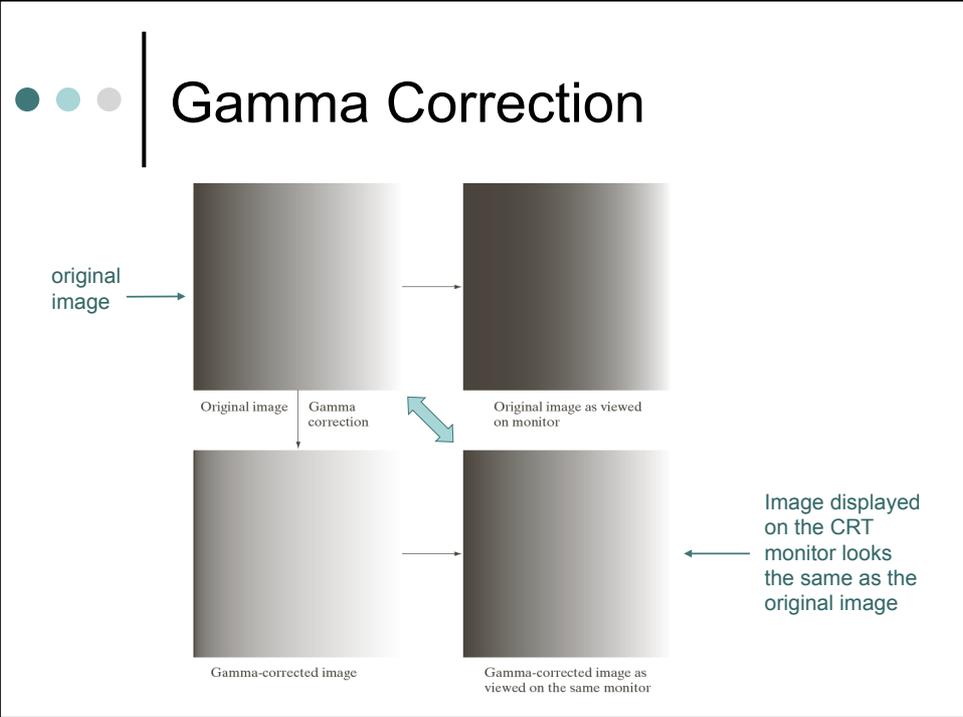
In particular for a CRT:

$$S = r^{2.5}$$

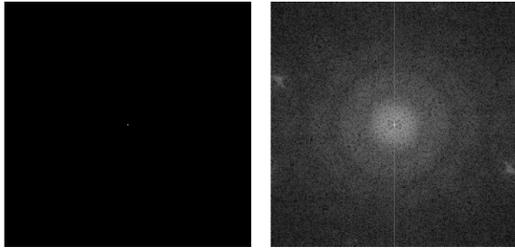
Gamma correction (preprocessing):

$$S = r^{0.4}$$

25



Log Transformation



(a) (b)

$$s = c \cdot \log(1 + r)$$

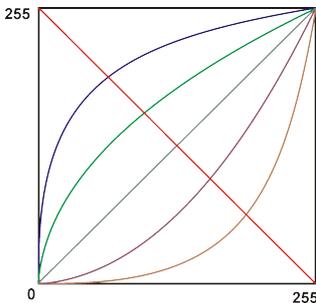
a) Dynamic range is [0, 28591].
 b) After log transform, the dynamic range becomes [0, 10.26], which is linearly scaled to [0,255]. Details become noticeable.

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Summary of Intensity Transformations

- Linear (inc. negative)
- Piecewise linear (e.g., gray level slicing)
- Nonlinear (e.g., gamma correction, Nth root, log, power, inverse log)

$s = T[r]$



- Identity
- Negative
- Log
- Inverse Log
- Power
- Nth root

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

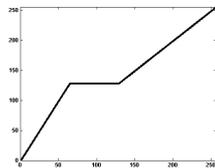


Using Look-Up Tables (LUTs)

For monochrome images with 256 gray levels, the LUT will be a 1D array of length 256.

Example:

$$s = \begin{cases} 2 \cdot f & \text{for } 0 < r \leq 64 \\ 128 & \text{for } 64 < r < 128 \\ f & \text{for } r \geq 128 \end{cases}$$



```
% Building the LUT
LUT = uint8(zeros([1 256]));
LUT(1:65) = 2*(0:64);
LUT(66:129) = 128;
LUT(130:256) = (130:256)-1;
% Applying the LUT
B = intlut(A, LUT)
```



Nonlinear Transformation Using LUTs

```
% Without a LUT
I = imread('klcc_gray.png');
tic
I2 = double(I);
J = 5*sqrt(I2);
O = uint8(J);
```

```
% With a LUT
I = imread('klcc_gray.png');
tic
LUT = double(zeros([1 256]));
LUT(1:256) = 5 * sqrt(0:255);
LUT_int8 = uint8(LUT);
O = intlut(I, LUT_int8);
```



Image Histogram

31



Questions

- What is the histogram of an image?
- How can the histogram be computed?
- What is histogram equalization?
- How can the image be modified to match a given histogram?



What is a histogram?

- The histogram of a monochrome image is a representation of the **frequency of occurrence of each intensity level** in the image.
- The data structure that stores the frequency values is a 1D array of numerical values, $h(r)$, whose individual elements store the number (or percentage) of image pixels that correspond to each possible intensity level r .



Histogram of an Image

$$h(r) = n$$

r : image intensity value (0,1,...,255)

n : total number (0,1,...,MN-1) of pixels in the image having the intensity value r

$$\sum_{i,j} h(r) = MN$$

$$f(r) = \frac{1}{MN} h(r)$$

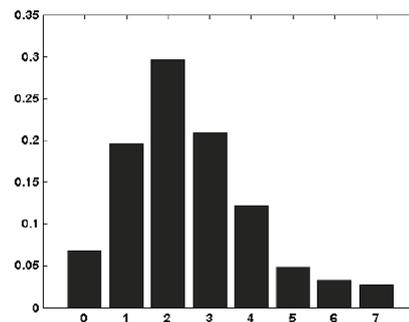
← Probability density function (pdf) of image intensities



Histogram Example

- Histogram for a hypothetical image containing 128×128 pixels and 8 intensity levels.

Gray level (r_k)	n_k	$p(r_k)$
0	1120	0.068
1	3214	0.196
2	4850	0.296
3	3425	0.209
4	1995	0.122
5	784	0.048
6	541	0.033
7	455	0.028
Total	16384	1.000



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Computing Image Histograms

- An array of B elements is created, where B is the number of histogram bins (or buckets). $B \leq K$, where K is the number of gray levels in the image. It is common to choose $B=K$.
- The histogram (number of pixels in each bin) is calculated as:

$$h(j) = \text{card}\{(x, y) | r_j \leq f(x, y) < r_{j+1}\}, \text{ for } 0 \leq j \leq B$$

$$r_j = j \cdot \frac{K}{B}$$

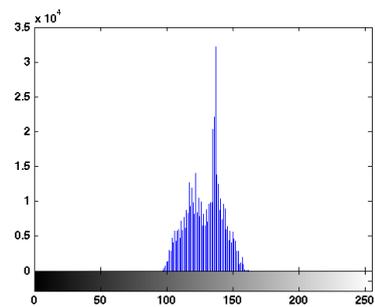


Example: Low Contrast Image



Pixels are grouped around intermediate gray-level values, indicating an image with low contrast

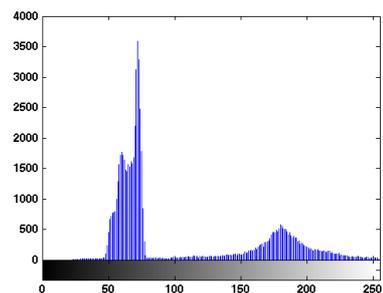
○ In MATLAB:
imhist



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



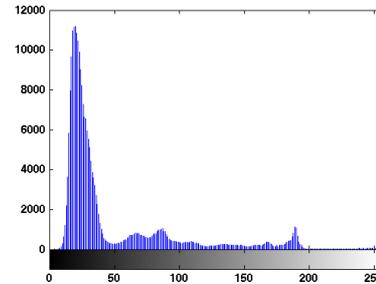
Example: Bimodal Image



- Two distinct hills.
- Image has high contrast since the two modes are well separated from each other.

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

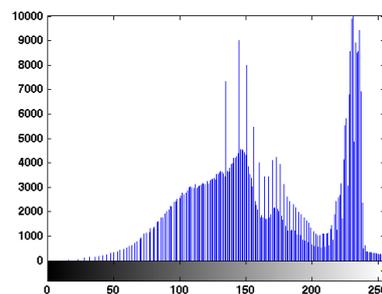
Example: Dark Image



- Histogram is concentrated in lower gray levels, which corresponds to a mostly dark image.

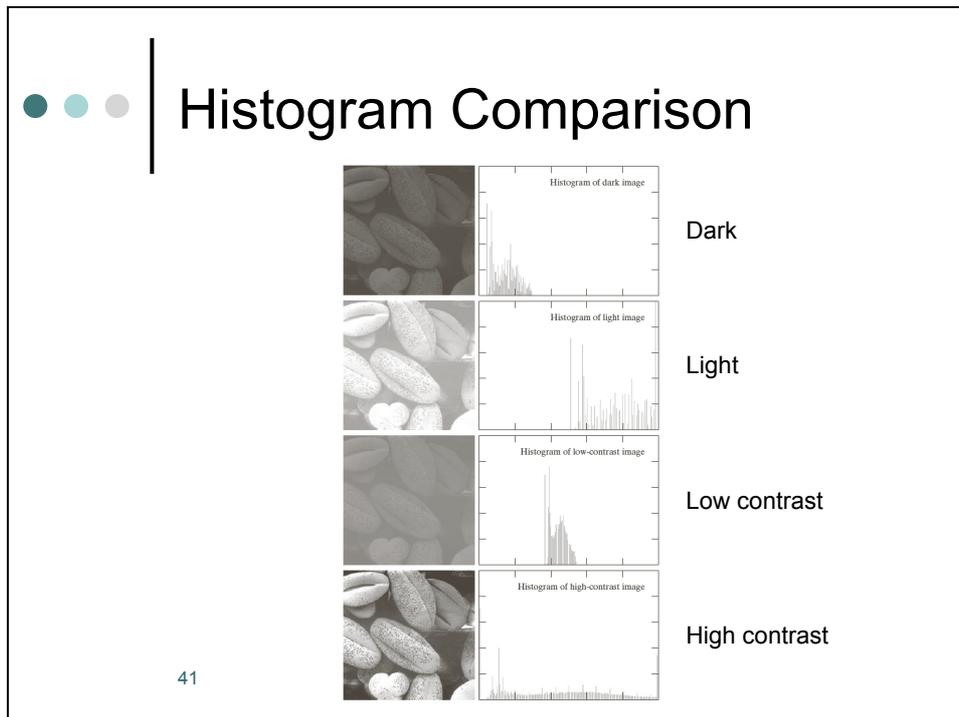
By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Example: Bright Image



- Histogram is grouped close to the higher gray-levels, which corresponds to a bright image.

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Utilization of Histograms

- Contemporary digital cameras have an optional real-time histogram overlay in their viewfinder.
- This information prevents taking underexposed or overexposed pictures.

Example:
Too dark → increase exposure



Summary

- Histograms provide a **statistical** representation of the intensity distribution in an image.
- Histograms can be used to evaluate image attributes such as minimum, average, and maximum intensity values, overall contrast and average brightness, and dominance of bright or dark pixels.
- Histograms do **not** contain any information about the spatial distribution of the pixels.
- Histograms can be **modified** to enhance the appearance of an image.



Histogram Modification Techniques



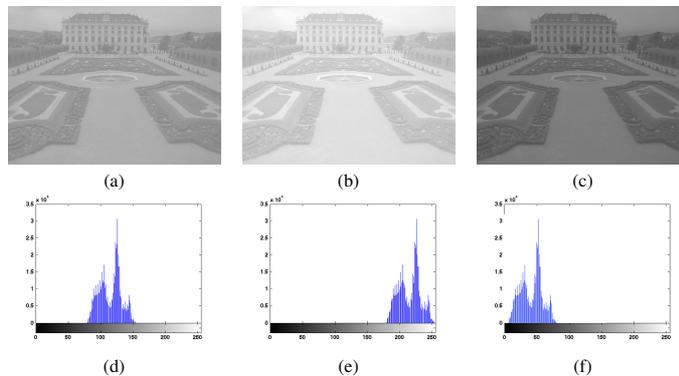
Histogram Modification Techniques

- Histogram sliding
- Histogram stretching
- Histogram shrinking
- Histogram equalization
- Histogram mathing
- Adaptive histogram equalization



Histogram Sliding

- Same as adding or subtracting a constant value (brightness change)



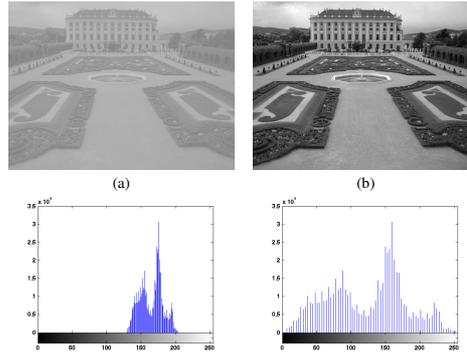
By Oge Marques

Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Histogram Stretching

- Expand the part of the histogram such that its nonzero intensity range occupies the full dynamic gray scale. (same as autocontrast)

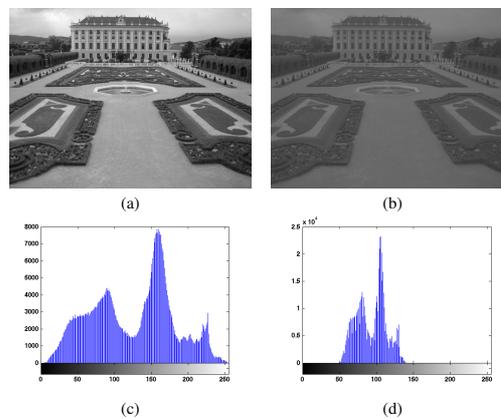
$$s = \frac{r - r_{min}}{r_{max} - r_{min}} \cdot (L - 1)$$



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Histogram Shrinking

- Compress the dynamic range $s = \left[\frac{s_{max} - s_{min}}{r_{max} - r_{min}} \right] (r - r_{min}) + s_{min}$



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Histogram Equalization

49



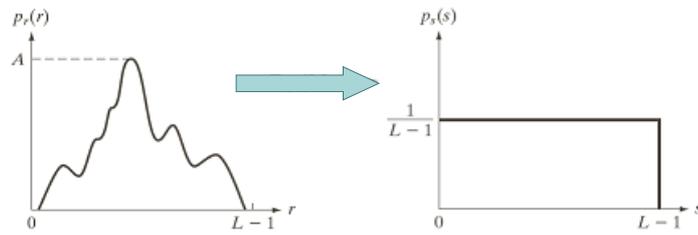
Histogram Equalization

- Map the pixel intensities in an image so as to obtain a uniform (flat) resulting histogram, in which the percentage of pixels of every intensity level is the same.



Histogram Equalization

Find an intensity mapping $s=T(r)$ such that the new intensity values have a flat probability density function (pdf):



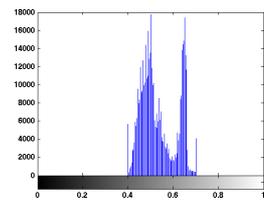
51



Histogram equalization



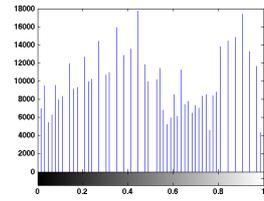
(a)



(b)



(c)



(d)

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

● ● ● | MATLAB: `histeq`

```
I = imread('sydney_low_contrast.png');  
I = rgb2gray(I);  
J = histeq(I);
```

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

● ● ● | pdf vs. cdf

The left graph shows the probability density function $f_r(x)$ plotted against x . The x-axis ranges from 0 to 255, and the y-axis has a tick mark at A . The curve is jagged and peaks at A .

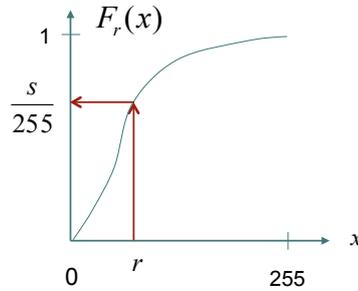
The right graph shows the cumulative distribution function $F_r(x)$ plotted against x . The x-axis ranges from 0 to 255, and the y-axis has a tick mark at 1. The curve is smooth and S-shaped, starting at (0,0) and ending at (255,1).

54



Finding s Given r

1. Calculate the histogram $h(r)$ of the image
2. Evaluate the cdf $F_r(x)$
3. Use the cdf to map r to s

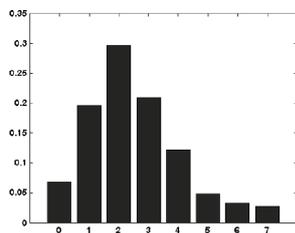


$$T(x) = 255 \cdot F_r(x)$$

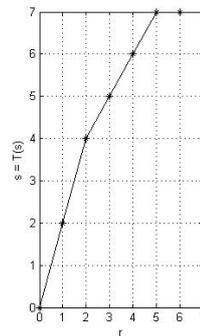
55



Example: Histogram Equalization



Gray level (r_k)	n_k	$p(r_k)$
0	1120	0.068
1	3214	0.196
2	4850	0.296
3	3425	0.209
4	1995	0.122
5	784	0.048
6	541	0.033
7	455	0.028
Total	16384	1.000



$$s_i = T(r_i) = \text{nearest integer} \left[7 \times \sum_{j=0}^i p(r_j) \right]$$

Example (continued)

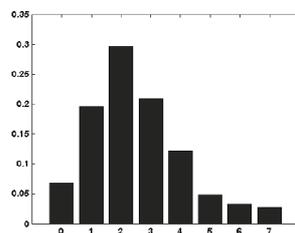
- Find the cdf

Gray level (r_k)	n_k	$p(r_k)$	
0	1120	0.068	$s_0 = T(r_0) = \sum_{j=0}^0 p(r_j) = p(r_0) = 0.068$
1	3214	0.196	$s_1 = T(r_1) = \sum_{j=0}^1 p(r_j) = p(r_0) + p(r_1) = 0.068 + 0.196 = 0.264 \rightarrow 2$
2	4850	0.296	
3	3425	0.209	
4	1995	0.122	$s_2 = \sum_{j=0}^2 p(r_j) = 0.560 \times 7 = 3.92 \approx 4,$
5	784	0.048	
6	541	0.033	$s_3 = 0.769 \times 7 = 5.383 \approx 5, s_4 = 0.891 \times 7 = 6.237 \approx 6,$
7	455	0.028	
Total	16384	1.000	$s_5 = 0.939 \rightarrow 7, s_6 = 0.972 \rightarrow 7, s_7 = 1 \rightarrow 7$

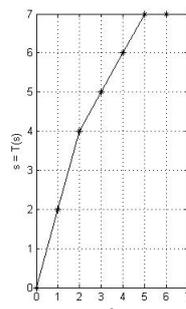
- Since the image was quantized with eight gray levels, multiply each s_k with 7 and round to the nearest integer:

$$s_0 \approx 0, s_1 \approx 2, s_2 \approx 4, s_3 \approx 5, s_4 \approx 6, s_5 \approx 7, s_6 \approx 7, s_7 \approx 7$$

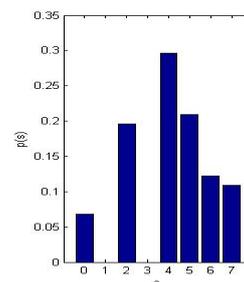
Example (Continued)



Original histogram



Transformation function



Equalized histogram

The equalized histogram is not perfectly flat, but it is the best possible result for this particular image.



Derivation

Let r and s denote original and transformed pixel values.
For an 8-bit image, we have $0 \leq r, s \leq 255$.

Find the transformation function $s=T(r)$ such that

$$f_s(x) = \text{constant} \quad \Rightarrow \quad F_s(x) = \frac{x}{255}$$

where $f(\cdot)$ denotes the probability density function (pdf) and F denotes the cumulative distribution function (cdf).

$$F_s(x) = P(s \leq x) = P(T(r) \leq x) = P(r \leq T^{-1}(x)) = F_r(T^{-1}(x)) = \frac{x}{255}$$

59



$T(\cdot)$?

$$F_s(x) = F_r(T^{-1}(x)) = \frac{x}{255}$$

$$F_r^{-1}\left(\frac{x}{255}\right) = T^{-1}(x) = y$$

$$F_r(y) = \frac{x}{255}$$

$$x = T(y)$$

$$T(y) = 255 \times F_r(y)$$

60



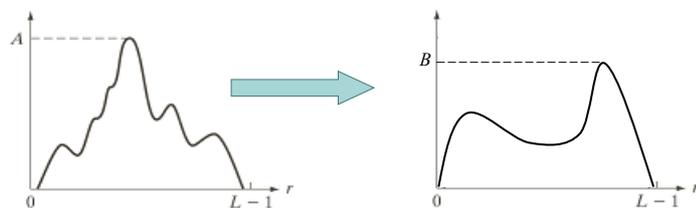
Histogram Matching

61



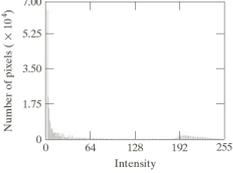
Histogram Matching

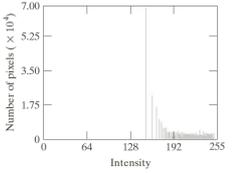
Find an intensity mapping $s = T(r)$ such that the new intensity values have an arbitrarily different probability density function:



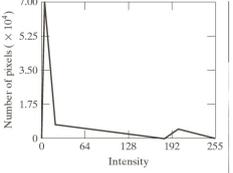
62

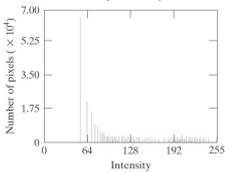
Histogram Equalization vs. Histogram Matching



Specified histogram





63

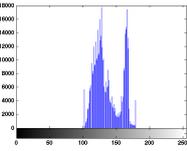
Histogram Specification



(a)



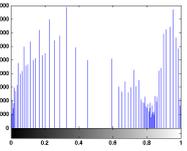
(b)



(c)



(d)



(e)

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



MATLAB: histeq

```

I = imread('sydney_low_contrast.png');
Id = im2double(I);
figure, imhist(Id), ylim('auto'), ...
      title ('Original histogram');

des_hist = uint8(zeros(1,256));
des_hist(1:128) = linspace(256,0,128);
des_hist(129:end) = linspace(0,256,128);
x_axis = 0:255;
figure, bar(x_axis, des_hist), axis tight, ...
      title('Desired histogram');

hgram = im2double(des_hist);
Jd = histeq(Id,hgram);
figure, imhist(Jd), ylim('auto'), ...
      title ('Resulting histogram');

```

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



$T(\cdot)$?

$$F_s(x) = F_r(T^{-1}(x))$$



$$F_r^{-1}(F_s(x)) = T^{-1}(x)$$

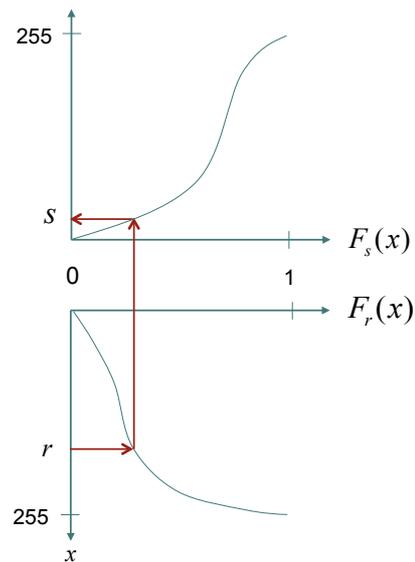


$$T(\underbrace{F_r^{-1}(F_s(x))}_r) = x$$

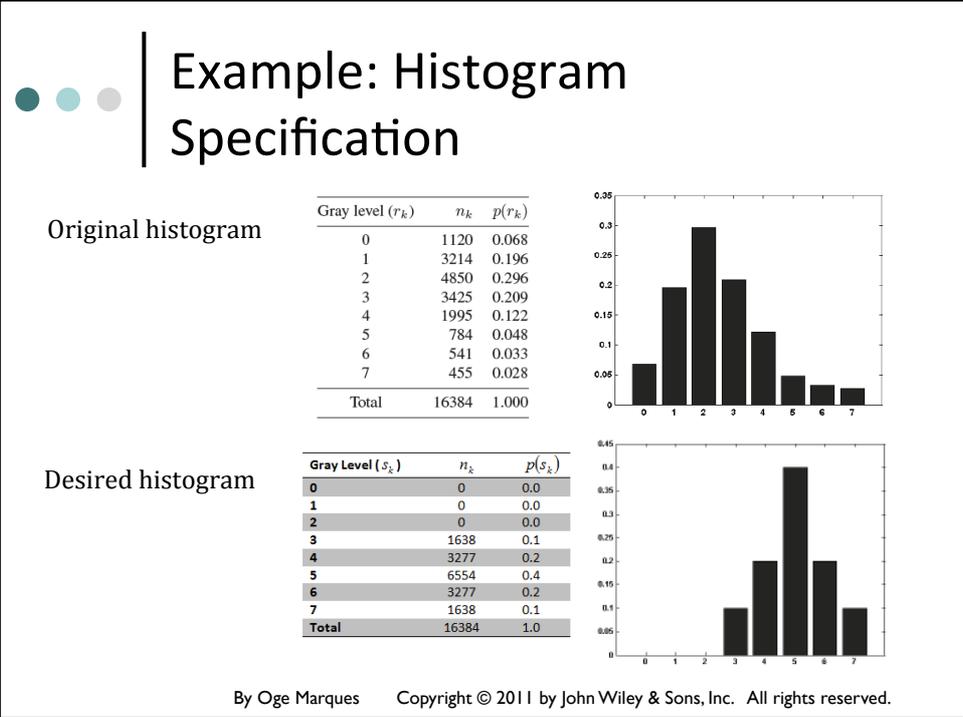
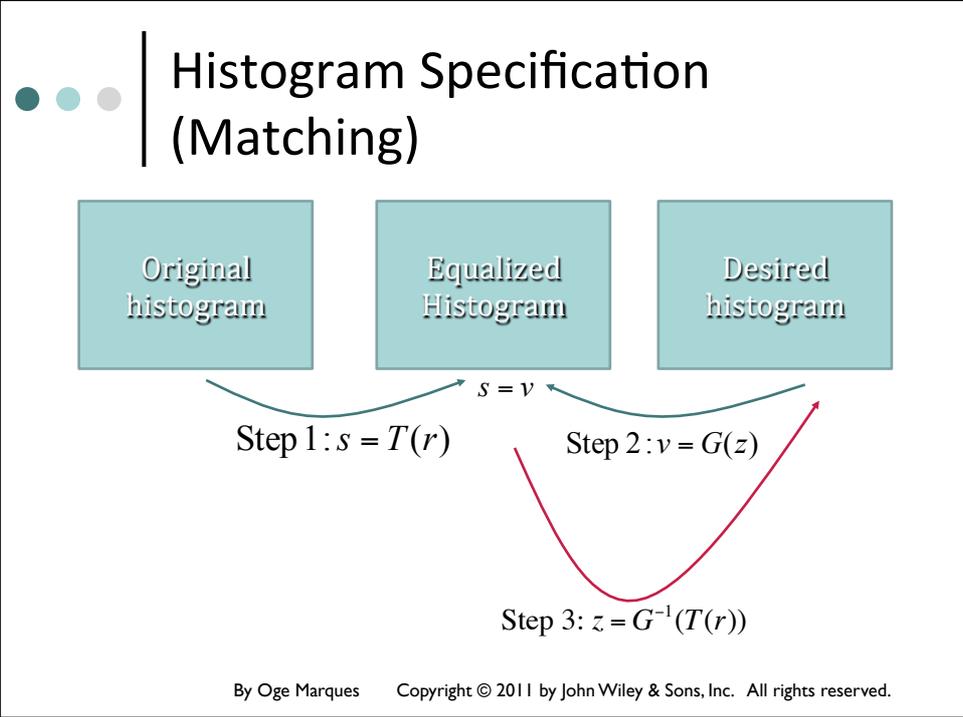
s (pointing to $F_s(x)$)
 r (pointing to $F_r^{-1}(F_s(x))$)



$$s = F_s^{-1}(F_r(r))$$



66





Example (continued)

- 1. Histogram equalized image has been found before:

$$s_0 \cong 0, s_1 \cong 2, s_2 \cong 4, s_3 \cong 5, s_4 \cong 6, s_5 \cong 7, s_6 \cong 7, s_7 \cong 7$$

- 2. Obtain the cdf of the desired probability mass function:

$$v_k = G(z_k) = \sum_{j=0}^k p(z_j)$$

$$v_0 = 0, v_1 = 0, v_2 = 0, v_3 = 0.1, v_4 = 0.3, v_5 = 0.7, v_6 = 0.9, v_7 = 1$$



Example (Continued)

- 3. Apply the inverse transformation function:

$$z = G^{-1}(s)$$

Search for each value of s_k , the closest value of v_k :

E.g., for $s_1 = 2/7 \cong 0.286$, the closest value of v_k is $v_4 = G(z_4) = 0.3$.

In inverse function notation $z_4 = G^{-1}(0.3)$

$$s_0 \cong 0, s_1 \cong 2, s_2 \cong 4, s_3 \cong 5, s_4 \cong 6, s_5 \cong 7, s_6 \cong 7, s_7 \cong 7$$

$$v_0 = 0, v_1 = 0, v_2 = 0, v_3 = 0.1, v_4 = 0.3, v_5 = 0.7, v_6 = 0.9, v_7 = 1$$



Example (Continued)

- The remaining values of s_k are mapped:

$$s_0 = 0 \rightarrow z_2$$

$$s_1 = 2/7 \cong 0.286 \rightarrow z_4$$

$$s_2 = 4/7 \cong 0.571 \rightarrow z_5$$

$$s_3 = 5/7 \cong 0.714 \rightarrow z_5$$

$$s_4 = 6/7 \cong 0.857 \rightarrow z_6$$

$$s_5 = 1 \rightarrow z_7$$

$$s_6 = 1 \rightarrow z_7$$

$$s_7 = 1 \rightarrow z_7$$

Matched Histogram

$$p(z_0) = 0.0$$

$$p(z_1) = 0.0$$

$$p(z_2) = 0.0$$

$$p(z_3) = 0.1$$

$$p(z_4) = 0.2$$

$$p(z_5) = 0.4$$

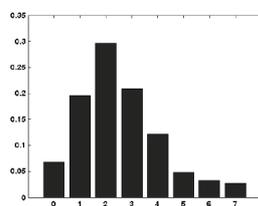
$$p(z_6) = 0.2$$

$$p(z_7) = 0.1$$

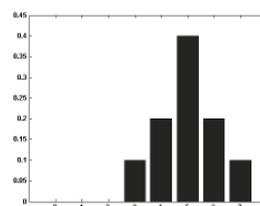


Direct Histogram Specification

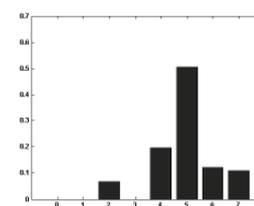
Original



Desired



Result



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Adaptive (Local) Histogram Equalization

73



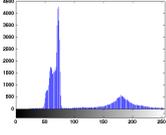
Local Histogram Equalization

- Use a sliding (rectangular/square) **window**, which moves across the image.
- Compute the histogram in the window and compute the mapping function.
- Map the **center** pixel of the window using the mapping function.
- Much more computationally expensive than global histogram equalization.

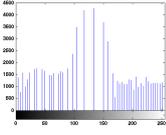
By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Global vs. Local Histogram Equalization

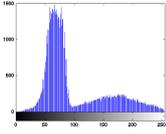
Original image and its histogram

After global histogram equalization

After local histogram equalization. Bimodal nature of the histogram is preserved while still improving the contrast.

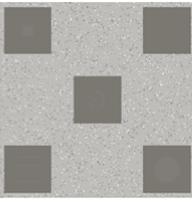



By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

Adaptive (Local) Histogram Equalization



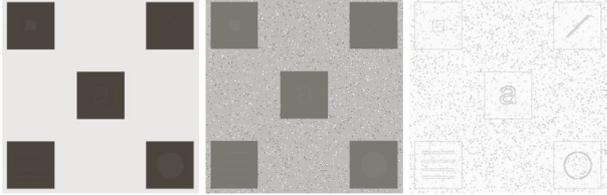
Original



Global HE

76

● ● ● | Adaptive (Local) Histogram Equalization



Original Global HE Local HE

77

The image shows three grayscale versions of a document page. The first, labeled 'Original', is very dark with high contrast. The second, 'Global HE', is a uniform, medium-gray image where all details are lost. The third, 'Local HE', is a bright, high-contrast image where details are preserved across the entire page.

● ● ● |

Neighborhood Processing

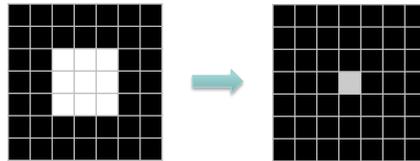
78



Neighborhood Operations

- Transformation depends on the value of the pixel as well as the values of its neighbors.

$$r(x, y) = T[s(x, y) : (x, y) \in W]$$



Neighborhood processing

- Main steps:
 - Define a reference point in the input image, $f(x_0, y_0)$.
 - Perform an operation that involves only pixels within a neighborhood around the reference point in the input image.
 - Apply the result of that operation to the pixel of same coordinates in the output image, $g(x_0, y_0)$.
 - Repeat the process for every pixel in the input image.

Convolution

- Convolution is a widely used linear operator that processes an image by computing -- for each pixel -- a weighted sum of the values of that pixel and its neighbors.
 - Depending on the choice of weights a wide variety of image processing operations can be implemented.

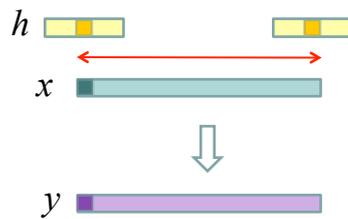
Convolution

input image I output image F

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

1-D Convolution

$$y[m] = h[m] * x[m] = \sum_i h[m-i]x[i]$$



83

Example

- Convolve $A = \{0,1,2,3,2,1,0\}$ and $B = \{1,3,-1\}$
 - Mirror array B and align its center (reference) value with the first (leftmost) value of array A.

A	0	1	2	3	2	1	0
B	-1	3	1				
A*B	1						

A	0	1	2	3	2	1	0
B				-1	3	1	
A*B	1	5	8	9			

A	0	1	2	3	2	1	0
B		-1	3	1			
A*B	1	5					

A	0	1	2	3	2	1	0
B					-1	3	1
A*B	1	5	8	9	4		

A	0	1	2	3	2	1	0
B			-1	3	1		
A*B	1	5	8				

A	0	1	2	3	2	1	0	
B						-1	3	1
A*B	1	5	8	9	4	1	-1	

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.

2-D Convolution

$$y[m,n] = \sum_i \sum_j h[m-i,n-j]x[i,j]$$

The diagram illustrates the convolution process. On the left, an input image x of size $M \times N$ is shown with a yellow kernel h of size $K \times L$ overlaid. An arrow points to the right, where the resulting output image y of size $(M + K - 1) \times (N + L - 1)$ is shown. A small green square in the bottom right corner of the output image represents the result of the convolution at that position.

85

Dealing with image borders

The diagram shows a gray square representing an image. A dashed line indicates the path of a kernel as it moves across the image. The top-left corner is shaded with diagonal lines and labeled "no coverage", indicating that the kernel cannot be applied there because it would go outside the image boundaries. The bottom-right corner is labeled "full coverage", indicating that the kernel can be applied there without any loss of information.

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



Dealing with image borders:

1. Keep the pixel values that cannot be reached by the overlapping mask untouched
 2. Pad the input image with zeros.
 3. Pad with extended (same) values.
 4. Pad with mirrored values.
 5. Treat the input image as a 2D periodic function whose values repeat themselves in both horizontal and vertical directions.
- o In MATLAB: check the `boundary_options` parameter for function `imfilter`.

By Oge Marques Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved.



o Convolve A and B:

$$A = \begin{bmatrix} 5 & 8 & 3 & 4 & 6 & 2 & 3 & 7 \\ 3 & 2 & 1 & 1 & 9 & 5 & 1 & 0 \\ 0 & 9 & 5 & 3 & 0 & 4 & 8 & 3 \\ 4 & 2 & 7 & 2 & 1 & 9 & 0 & 6 \\ 9 & 7 & 9 & 8 & 0 & 4 & 2 & 4 \\ 5 & 2 & 1 & 8 & 4 & 1 & 0 & 9 \\ 1 & 8 & 5 & 4 & 9 & 2 & 3 & 8 \\ 3 & 7 & 1 & 2 & 3 & 4 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

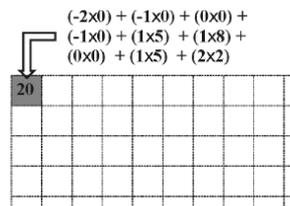
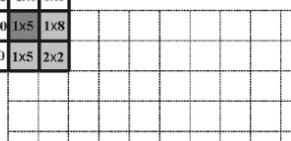
o Flipped B:

$$\begin{bmatrix} -2 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

o Result A*B:

$$\begin{bmatrix} 20 & 10 & 2 & 26 & 23 & 6 & 9 & 4 \\ 18 & 1 & -8 & 2 & 7 & 3 & 3 & -11 \\ 14 & 22 & 5 & -1 & 9 & -2 & 8 & -1 \\ 29 & 21 & 9 & -9 & 10 & 12 & -9 & -9 \\ 21 & 1 & 16 & -1 & -3 & -4 & 2 & 5 \\ 15 & -9 & -3 & 7 & -6 & 1 & 17 & 9 \\ 21 & 9 & 1 & 6 & -2 & -1 & 23 & 2 \\ 9 & -5 & -25 & -10 & -12 & -15 & -1 & -12 \end{bmatrix}$$

-2x0	-1x0	0x0
-1x0	1x5	1x8
0x0	1x5	2x2





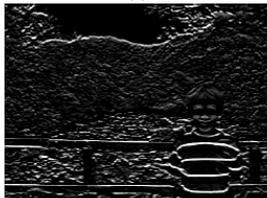
(a)



(b)



(c)



(d)

Low-pass filter	High-pass filter	Horizontal edge detection
$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Separable 2-D Filter

$$y[m, n] = \sum_i \sum_j h[m-i, n-j] x[i, j]$$

separable \longrightarrow $h[m, n] = h_1[m] h_2[n]$

90



Implementation of a Separable 2-D Filter

$$\begin{aligned}
 y[m,n] &= \sum_i \sum_j h_1[m-i]h_2[n-j]x[i,j] \\
 &= \sum_j h_2[n-j] \underbrace{\sum_i h_1[m-i]x[i,j]}_{\leftarrow \text{rows}} \\
 &= \sum_j h_2[n-j] \bar{x}[m,j] \leftarrow \text{columns}
 \end{aligned}$$

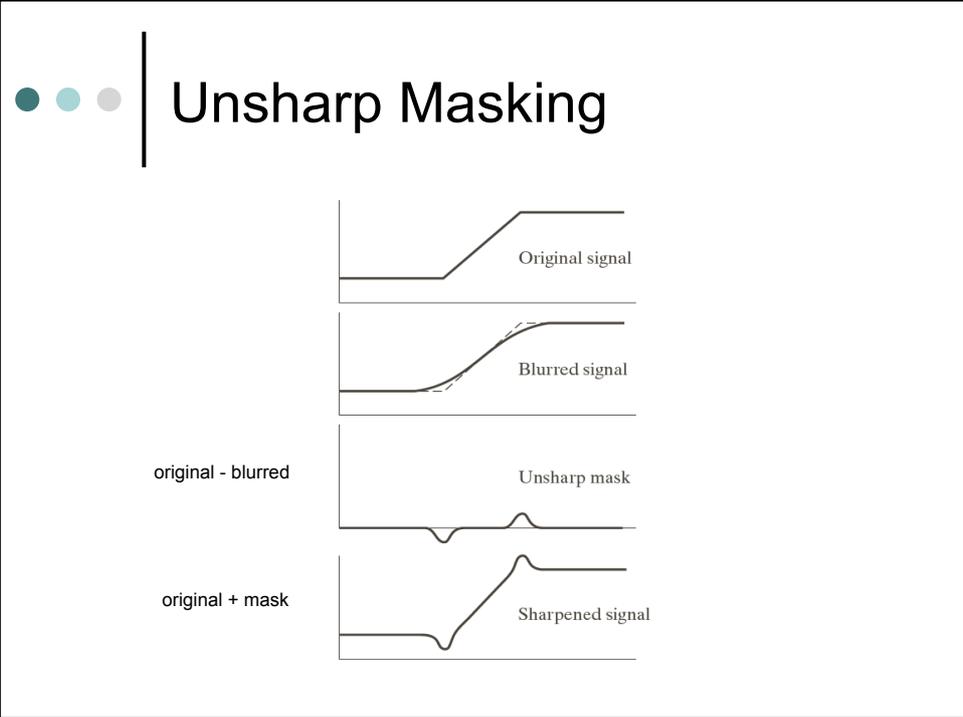
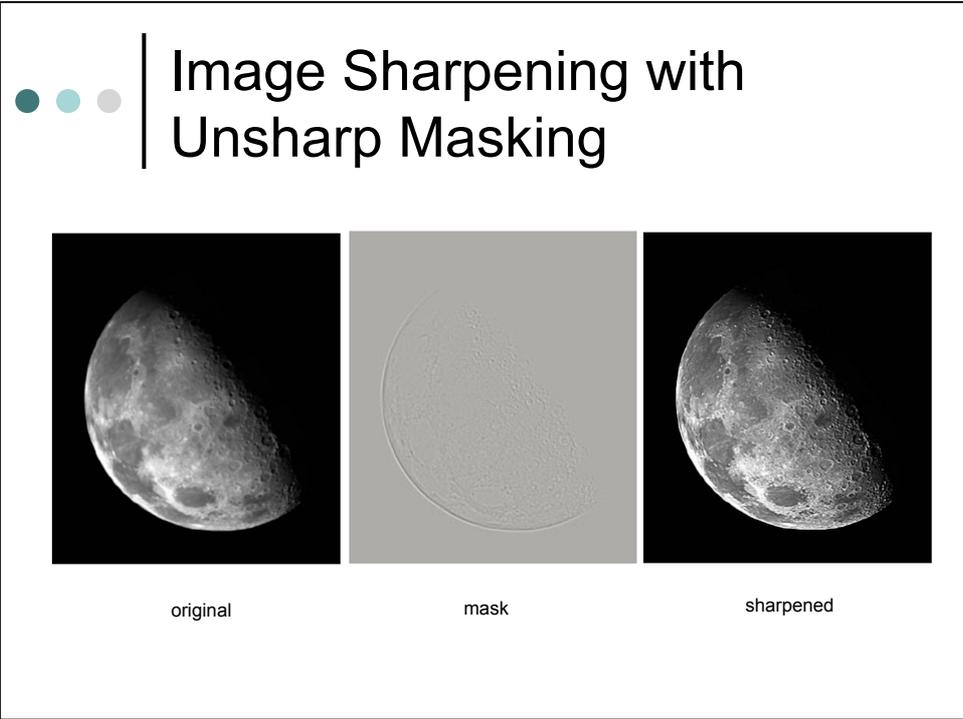
Two 1-D convolution computations: $2N \log N$
 Rather than one 2-D convolution: $N^2 \log N^2$

91



Edge Enhancement

92





Project 1.2

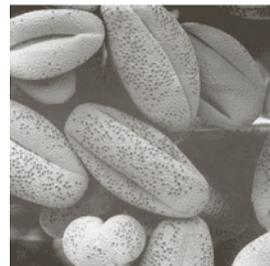
Image Enhancement

Due 10.10.2013

95



Histogram Equalization



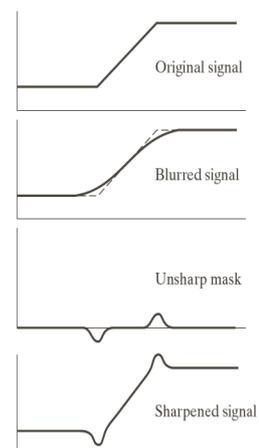
96

Problem 1: Histogram Equalization & Matching

1. Pick two images, one with low and the other with high exposure (i.e., a dark and a bright image).
2. Calculate and display the histogram of each image.
3. Apply histogram equalization to each image and display the resulting images and their histograms.
4. Modify the dark image so that its histogram matches that of the bright image. Display the modified image and its histogram.
5. Comment on the results.

97

Unsharp Masking



98



Problem 2: Unsharp Masking

1. Pick an unsharp image.
2. Calculate and display an unsharp mask for the image. You can use a 3x3 mean filter for this purpose.
3. Obtain and display the sharpened image.
4. Compare the original image with the image obtained in Step 3 and comment on any improvements.
5. If using a color image, implement Steps 2 and 3 on all three bands and then combine the sharpened bands.

99



Next Lecture

- COLOR THEORY

100